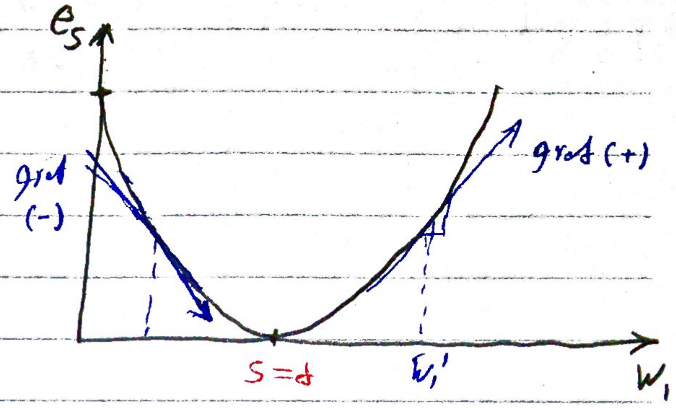
Back Propagation
« Gradient Desent »

$e = d - s$

$e_s = \frac{1}{2} e^2 = \frac{1}{2} (d-s)^2$



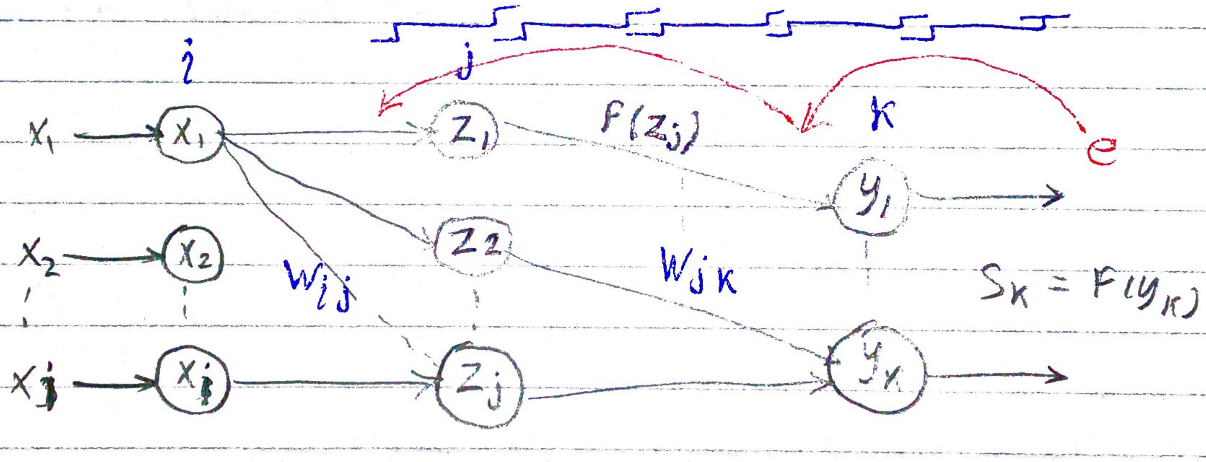
$\frac{\partial e_s}{w_1}$ = gradient

شرط استخدام Back propagation _

أن تكون Activation fn. قابلة للتفاضل _

$\Delta w \to$ positive | $\Delta w \to$ negative

$\Delta w \propto -$ grad



$\longrightarrow F \to S = \checkmark$

$\to e = d - s = \checkmark$

① Find the output for specific input

② Calculate the error

③ error back to previous Layer to change Weights

$S_K = F(y_K)$ | $Z_j = \sum_i W_{ij} \, X_i$

$y_K = \sum_j W_{jK} \, F(z_j)$ | $E = \frac{1}{2} \sum_K (d_K - F(y_K))^2$

Update Weights :

$$\frac{\partial E}{\partial w_{jk}} = -\frac{2}{2}\,(d_k - f(y_k))\,\frac{\partial f(y_k)}{\partial w_{jk}}$$ ((For output layer))

$$= -e\,\frac{\partial f(y_k)}{\partial w_{jk}}$$

$$= -e\,\underbrace{\frac{\partial f(y_k)}{\partial y_k}}_{\text{depends on Activation Function}} \cdot \frac{\partial y_k}{\partial w_{jk}}$$

$$= -e\,\frac{\partial f(y_k)}{\partial y_k}\, f(z_j)$$

$$\delta_k = e\,\frac{\partial f(y_k)}{\partial y_k}$$

For sigmoidal function

$$= -e\, f(y_k)\,(1 - f(y_k))\, f(z_j)$$

$$\delta_k = e\, f(y_k)\,(1 - f(y_k))$$

$$\frac{\partial E}{\partial w_{jk}} = -\,\delta_k\, f(z_j)$$

For weight from input to hidden layer

$$\frac{\partial E}{\partial w_{ij}} = \sum_k -\frac{2}{2}\,(d_k - f(y_k))\,\frac{\partial f(y_k)}{\partial w_{ij}}$$

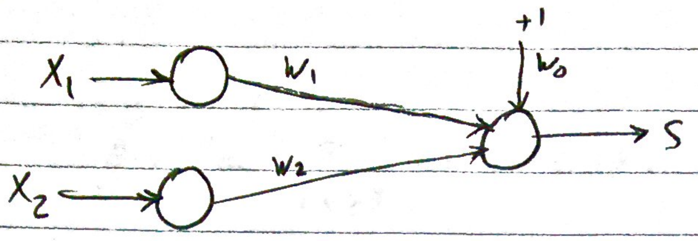
$$= \sum_k -\,\underbrace{e\,\frac{\partial f(y_k)}{\partial y_k}}_{\delta_k} \cdot \underbrace{\frac{\partial y_k}{\partial f(z_j)}}_{w_{jk}} \cdot \underbrace{\frac{\partial f(z_j)}{\partial z_j}}_{(f(z_j)(1-f(z_j)))} \cdot \underbrace{\frac{\partial z_j}{\partial w_{ij}}}_{x_i}$$

$$= \sum_k -\,\delta_k \quad w_{jk} \quad \underbrace{(f(z_j)\,(1-f(z_j)))}_{\text{for sigmoid}} \quad x_i$$

$$= -\underbrace{\left(\sum_k \delta_k w_{jk}\right) f(z_j)\,(1 - f(z_j))}_{\delta_j} \quad x_i$$

$$= -\,\delta_j\, x_i$$

1



$w_1 = -0.3$ $\qquad$ $w_2 = 0.3$ $\qquad$ $w_0 = 0.6$

$d = 0.9$ $\qquad$ $X_1 = 3.1$ $\qquad$ $X_2 = 2.7$ $\qquad$ Sigmoidal

a) Find e, E

b) Find gradient for w_0, w_1, w_2

a) $S = f(y) = ?$

$$y = X_1 w_1 + X_2 w_2 + w_0 = 0.4$$

$$S = F(y) = \frac{1}{1 + e^{-y}} = 0.599$$

$$e = d - s = 0.9 - 0.599 = 0.301$$

$$E = \frac{1}{2}(d-s)^2 = \frac{1}{2} e^2 = 0.045$$

b) $$\frac{\partial E}{\partial w_i} = \underbrace{\frac{\partial E}{\partial f(y)} \cdot \frac{\partial f(y)}{\partial y}}_{-\delta_K} \cdot \underbrace{\frac{\partial y}{\partial w_i}}_{X_i}$$

$$\delta_K = e f(y)(1 - f(y)) = 0.072$$

$$\frac{\partial E}{\partial w_0} = -\delta_K = -0.072$$

$$\frac{\partial E}{\partial w_1} = -\delta_K X_1 = 0.229$$

$$\frac{\partial E}{\partial w_1} = -\delta_K X_2 = -0.194$$

2 For Bipolar Sigmoidal from 1

$$y = 0.4$$

$$g(y) = \frac{2}{1+e^{-y}} - 1 = 0.197$$

$$\frac{\partial E}{\partial w_i} = \frac{\partial E}{\partial g(y)} \cdot \frac{\partial g(y)}{\partial y} \cdot \frac{\partial y}{\partial w_i}$$

$$\delta_K = e \cdot \frac{1}{2} (1 - g(y)^2) = 0.338$$

$$\frac{\partial E}{\partial w_0} = - \delta_K = -0.338$$

$$\frac{\partial E}{\partial w_1} = - \delta_K \; X_1 = -1.048$$

$$\frac{\partial E}{\partial w_2} = - \delta_K \; X_2 = -0.913$$

3 From 1

$$\frac{\partial E}{\partial w_0} = -0.13 \qquad \frac{\partial E}{\partial w_1} = -0.52 \qquad \frac{\partial E}{\partial w_2} = -0.65$$

Find X_1, X_2 $\quad w_0 = 0.6 \quad w_1 = -0.5 \quad w_2 = 0.5$

$\delta = 0.9$

$$\frac{\partial E}{\partial w_0} = - \delta_K = -0.13$$

$$\delta_K = 0.13$$

$$\frac{\partial E}{\partial w_1} = - \delta_K \; X_1$$

$$-0.13 \; X_1 = -0.52 \rightarrow X_1 = 4$$

$$\frac{\partial E}{\partial w_2} = - \delta_K \; X_2 \Rightarrow \quad X_2 = 5$$

$\boxed{4}$ frm ③

$W_1 = 0.5$ $\quad W_2 = -0.5$ $\quad e = 0.52$

find W_0

$\delta_k = 0.13$

Sigmidal

~~e — d = s~~

~~s — d - e =~~

$$\delta = e \quad f(y) \quad (1 - f(y))$$

$$0.13 = 0.52 \; (S - S^2)$$

$$\frac{0.13}{0.52} = S - S^2$$

$$S^2 - S + 0.25 = 0$$

$$(S - 0.5)^2 = 0$$

$$S = f(y) = 0.5$$

$$y = \ln \frac{f(y)}{1 - f(y)} = 0$$

$$y = w_1 \; x_1 + w_2 \; x_2 + w_0$$

$$0 = 0.5 * 4 - 0.5 * 5 + w_0$$

$$\boxed{W_0 = 0.5}$$

$\boxed{6}$ repeat (4) $\rightarrow e = 0.5$

$$F(y)\,(1 - F(y)) = \frac{\delta}{e} = \frac{0.13}{0.5} = 0.26$$

$$F(y) = \frac{1 \pm \sqrt{1 - 4\,(0.26)}}{2}$$

no Solution

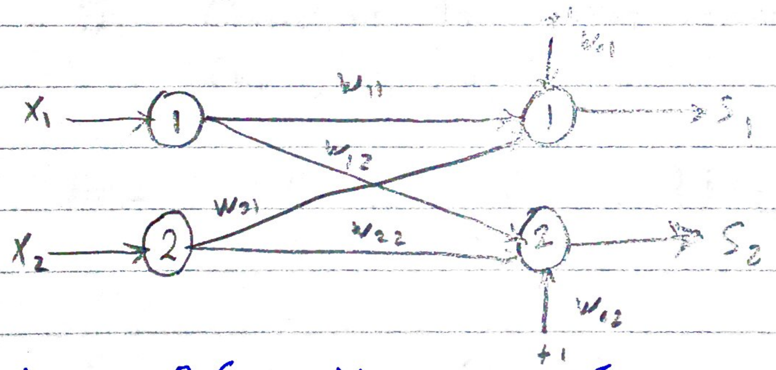
$$\frac{\delta}{e} \leq 0.25$$

$$e \geq \frac{0.13}{0.25}$$

$$e \geq 0.52$$

أقل Error يمكن الوصول إليه

$\boxed{7}$



$X_1 = -2.6$ $\quad W_{01} = 0.33$ $\quad W_{11} = 1.1$ $\quad W_{21} = -0.8$

$X_1 = -1.9$ $\quad W_{02} = -0.46$ $\quad W_{12} = -1.2$ $\quad W_{22} = 0.7$

$d_1 = 0.51$ $\qquad d_2 = 1.21$

Find a) e_1, e_2

b) gradient of all weights

a) $e_1 = d_1 - S_1 = 0.51 - S_1$

$S_1 = f(y_1)$

$y_1 = -0.99$

$S_1 = \frac{1}{1+e^{-y}} = 0.271$

$e_1 = 0.51 - 0.271 = 0.239$

$y_2 = 1.33$

$S_2 = 0.791$

$e_2 = 0.419$

$E = \frac{1}{2}\left[(d_1 - S_1)^2 + (d_2 - S_2)^2\right]$

$= \frac{1}{2}(e_1^2 + e_2^2)$

$= 0.116$

b) ~~δ_1~~ Connections to output neuron 1

(w_{01}, w_{11}, w_{21})

$$\frac{\partial E}{\partial w_{01}} = \frac{\partial E}{\partial f(y_1)} \cdot \frac{\partial f(y_1)}{\partial y_1} \cdot \frac{\partial y_1}{w_{01}} = -\delta_1 x_0$$

$\delta_1 = e_1 S_1 (1 - S_1) = 0.047$

$\frac{\partial E}{\partial w_{01}} = -\delta_1 = -0.047$

$\frac{\partial E}{\partial w_{11}} = -\delta_1 X_1 = 0.122$

$\frac{\partial E}{\partial w_{21}} = -\delta_1 X_2 = 0.089$

Connections to output neuron ②
(W_{02}, W_{12}, W_{22})

$\delta_2 = e_2 S_2(\cancel{1} - S_2) = 0.069$

$\frac{\partial E}{\partial W_{02}} = - \delta_2 = - 0.069$

$\frac{\partial E}{\partial W_{12}} = - \delta_2 X_1 = 0.179$

$\frac{\partial E}{\partial W_{22}} = - \delta_2 X_2 = 0.131$

$\boxed{8}$ Prm ⑦ Bipolar Sigmoid

$e_1 = 1.22$ $d_1 = 1.41$

$e_2 = -0.81$ $d_2 = -1.23$

$W_{01} = -0.53$ $W_{11} = -0.92$ $W_{21} = 0.76$

$W_{02} = 0.53$ $W_{12} = 0.87$ $W_{22} = -0.65$

a) Find X_1, X_2
b) Find gradient

a) $e_1 = d_1 - S_1 \longrightarrow S_1 = 0.19$

$e_2 = d_2 - S_2 \longrightarrow S_2 = -0.42$

$y_1 = \ln \frac{1+S_1}{1-S_1} = 0.385$

$y_2 = \ln \frac{1+S_2}{1-S_2} = -0.895$

$$y_1 = -0.92\, X_1 + 0.76\, X_2 - 0.53 = 0.385$$

$$0.92\, X_1 - 0.76\, X_2 = -0.915 \rightarrow ①$$

$$y_2 = 0.87\, X_1 - 0.65\, X_2 + 0.53 = -0.895$$

$$0.87\, X_1 - 0.65\, X_2 = -1.425 \rightarrow ②$$

* Solving ①, ②

$$X_1 = -7.686 \qquad X_2 = -8.101$$

b) gradient

* Connection to output ①
(W_{01}, W_{11}, W_{12})

$$\delta_1 = e_1 \cdot \frac{1}{2}(1 - s_1^2) = 0.588$$

$$\frac{\partial E}{\partial w_{01}} = -\delta_1 = -0.588$$

$$\frac{\partial E}{\partial w_{11}} = -\delta_1 X_1 = 4.519$$

$$\frac{\partial E}{\partial w_{12}} = -\delta_1 X_2 = 4.763$$

* Connection to output (2)

(w_{02}, w_{12}, w_{22})

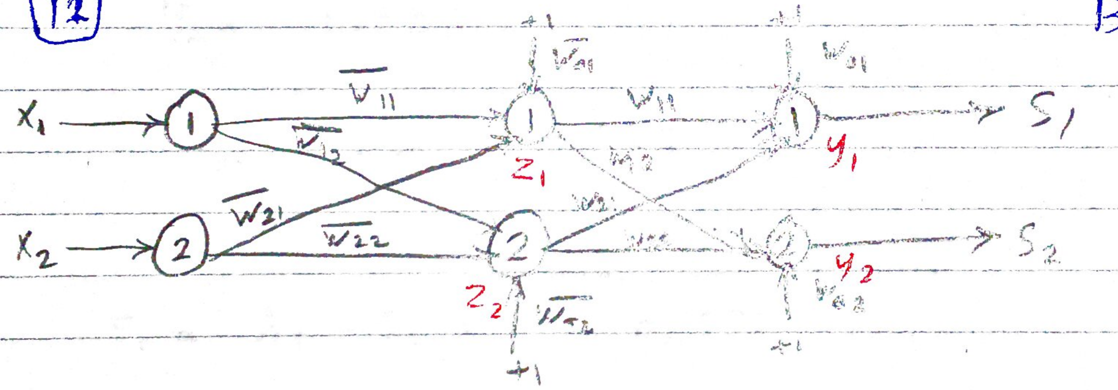
$$\delta_2 = e_2 \cdot \frac{1}{2}(1 - s_2^2) = 0.324$$

$$\frac{\partial E}{\partial w_{02}} = -\delta_2 = -0.324$$

$$\frac{\partial E}{\partial w_{12}} = -\delta_2 x_1 = -2.567$$

$$\frac{\partial E}{\partial w_{22}} = -\delta_2 x_2 = \cancel{2.567} \quad -2.706$$

$\boxed{12}$ Binary Sigmoidal



Weights ~~are~~ given, $x_1 = -3.2$ $x_2 = 2.8$

$d_1 = 0.18$ $d_2 = 0.11$

a) find e_1, e_2

b) find gradient

a) ~~for o/p~~ ~~(1)~~

for hidden (1) $\rightarrow z_1$

$z_1 = -1.164$

$F(z_1) = 0.238$

* For hidden ② z_2

$z_2 = 0.98$

$f(z_2) = 0.727$

* For output ① y_1

$y_1 = 1.103$

$f(y_1) = S_1 = 0.751$

* For output ② y_2

$y_2 = 1.003$

$f(y_2) = S_2 = -0.622$

$e_1 = d_1 - S_1 = -0.571$

$e_2 = d_2 - S_2 = -0.622$

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b) gradient

* For output neuron ① (w_{01}, w_{11}, w_{21})	* For output ② (w_{02}, w_{12}, w_{22})
$\delta_1 = e_1 S_1'(1-S_1) = -0.107$	$\delta_2 = e_2 S_2 (1-S_2) = -0.122$
$\frac{\partial E}{\partial w_{01}} = -\delta_1 = 0.107$	$\frac{\partial E}{\partial w_{02}} = -\delta_2 = 0.122$
$\frac{\partial E}{\partial w_{11}} = -\delta_1 f(x_1) = 0.25$	$\frac{\partial E}{\partial w_{12}} = -\delta_2 f(x_2) = 0.29$

$$\frac{\partial E}{\partial w_{21}} = -\delta_1 f(Z_2) = 0.078 \qquad \Bigg| \qquad \frac{\partial E}{\partial w_{22}} = -\delta_2 f(Z_2) = 0.089$$

* for hidden ①

$$\frac{\partial E}{\partial \bar{w}_{01}} = \left[\frac{\partial E}{\partial S_1} \cdot \frac{\partial S_1}{\partial y_1} \cdot \frac{\partial y_1}{\partial f(z_1)} \cdot \frac{\partial f(z_1)}{\partial z_1} \cdot \frac{\partial z_1}{\partial \bar{w}_{01}}\right]$$

$$+ \left[\frac{\partial E}{\partial S_2} \cdot \frac{\partial S_2}{\partial y_2} \cdot \frac{\partial y_2}{\partial f(z_1)} \cdot \frac{\partial f(z_1)}{\partial z_1} \cdot \frac{\partial z_1}{\partial \bar{w}_{01}}\right]$$

$$= \left[\underbrace{\frac{\partial E}{\partial S_1} \cdot \frac{\partial S_1}{\partial y_1}}_{-\delta_1} \cdot \underbrace{\frac{\partial y_1}{\partial f(z_1)}}_{w_{11}} + \underbrace{\frac{\partial E}{\partial S_2} \cdot \frac{\partial S_2}{\partial y_2}}_{-\delta_2} \cdot \underbrace{\frac{\partial y_2}{\partial f(z_1)}}_{w_{12}}\right] \frac{\partial f(z_1)}{\partial z_1} \cdot \underbrace{\frac{\partial z_1}{\partial \bar{w}_{01}}}_{X_0}$$

$$= \underbrace{-\left[\delta_1 w_{11} + \delta_2 w_{12}\right] f(z_1)(1-f(z_1))}_{\delta_I} X_0$$

$$\frac{\partial E}{\partial \bar{w}_{01}} = -\delta_I \cancel{x} = 0{,}029 \qquad \delta_I = -0{,}029$$

$$\frac{\partial E}{\partial \bar{w}_{11}} = -\delta_I X_1 = -0.093$$

$$\frac{\partial E}{\partial \bar{w}_{21}} = -\delta_I X_2 = 0.081$$

* for hidden 2

$$\overline{\delta_2} = [\delta_1 W_{21} + \delta_2 W_{22}] F(z_2)(1-F(z_2))$$

$$\overline{\delta_2} = -0.027$$

$$\frac{\partial E}{\partial \overline{W_{02}}} = -\overline{\delta_2} = \cancel{0.027}\ 0.027$$

$$\frac{\partial E}{\partial \overline{W_{12}}} = -\overline{\delta_2}\, X_1 = -0.086$$

$$\frac{\partial E}{\partial \overline{W_{22}}} = -\overline{\delta_2}\, X_2 = 0.076$$